Learning Material

Viscosity and Surface Tension

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Viscosity

- It is a measure of a fluid's resistance to flow.
- High viscous fluids flow in diffcult mannar.
- Ex: Honey, Engine oil, cooking oil, Lubricant, Liquid soap etc...

U • Liquids which experience difficulty while flowing → Viscous.

• The resistance experienced by a liquid during flow is due to the internal friction between its various layers, one moving over the other.

• The internal friction between fluid layers in motion is known as viscous drag.

Coefficient of viscosity:

The rate at which velocity of flow changes with distance from the fixed plane is known as velocity gradient.

If dv is the difference in velocity between two layers distance dy apart then velocity gradient $= \frac{dv}{dy}$

The viscous drag or viscous force per unit area (P)

$$P \propto \frac{dv}{dy}$$

$$= \eta \frac{dv}{dy}$$

$$\eta = \frac{P}{dv/dy}$$

η - coefficient of viscosity

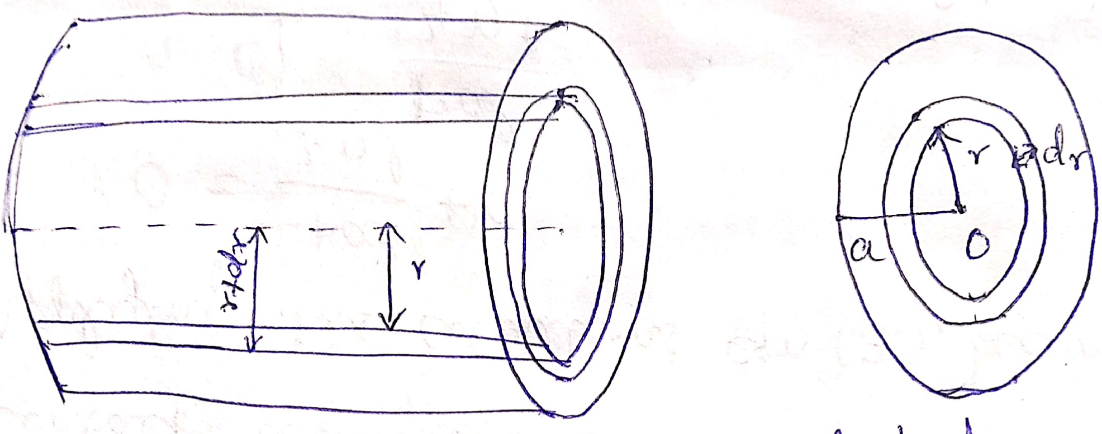
If $\frac{dv}{dy} = 1$, $\eta = P$. Hence coefficient of viscosity of a liquid is the tangential stress necessary to maintain unit velocity gradient in the liquid.

$$\eta = \frac{\text{Tangential stress}}{\text{velocity gradient}}$$

unit - N/m^2.

~~Pois~~

Poiseille's formula for liquid flow through a capillary tube:



P – const. pressure maintained between the two ends of a capillary tube.

l – length

a – radius

v – max. along the axis & zero near the walls of the tube.

(ie) there is no radial flow in the tube.

v – velocity on the inner surface

v – dv – " outer "

velocity gradient $= -\frac{dv}{dr}$

Surface Area of the shell, $A = 2\pi r l$

$$F_v = -\eta A \frac{dv}{dr} = -\eta 2\pi r l \frac{dv}{dr} \quad \text{——(1)}$$

Forward force

F_f = Pressure difference × Area of cross section of the inner cylinder

$F_f = P \times \pi r^2$ when the motion is steady —— (2)

The viscous dragging force = Forward driving force.

From eqn (1) & (2),

$$-\eta 2\pi r l \frac{dv}{dr} = P\pi r^2 \text{ (or)}$$

$$dv = \frac{-P}{2\eta l} r\,dr$$

Integrating we get, $v = \int \frac{-P}{2\eta l} r\,dr$

$$\text{or } v = \frac{-P}{2\eta l} \cdot \frac{r^2}{2} + C_1$$

$$= \frac{-Pr^2}{4\eta l} + C_1 \quad \text{—— (3)}$$

C_1 - const. of integration

$v = 0$ when $r = a$ because the layers in contact with the tubes are stationary.

Applying this conditions eqn (3), becomes,

$$0 = \frac{-Pa^2}{4\eta l} + C_1$$

$$C_1 = \frac{Pa^2}{4\eta l}$$

Sub. this value of C_1 in eqn (3),

$$v = \frac{Pa^2}{4\eta l} - \frac{Pr^2}{4\eta l}$$

$$= \frac{p}{4\eta l}(a^2 - r^2) \quad \text{——— (4)}$$

This gives the averages velocity of the liquid flowing through the cylindrical shell. Then

dV = Area of cross section of the shell × velocity of flow

$$= 2\pi r dr \times v$$

$$= 2\pi r dr \times \frac{p}{4\eta l}(a^2 - r^2)$$

$$= \frac{\pi p}{2\eta l}(a^2 r - r^3) dr \quad \text{——— (5)}$$

The volume of ~~the~~ liquid, V flowing through the entire cylinder / sec is obtained by integrating dV between the limits r = 0 & r = a.

$$V = \int_0^a \frac{\pi p}{2\eta l}(a^2 r - r^3) dr$$

$$= \frac{\pi p}{2\eta l}\left(a^2\frac{r^2}{2} - \frac{r^4}{4}\right)_0^a$$

$$= \frac{\pi p a^4}{2\eta l \times 4} = \frac{\pi p a^4}{8 l \eta} \quad \text{——— (6)}$$

If h - const. pressure head which maintains the flow, ρ the density of the liquid, and g the acceleration due to gravity, then $p = h\rho g$.

Sub. this value of p in eqn (6)

$$V = \frac{\pi h \rho g a^4}{8 l \eta} \quad \text{--- (7)}$$

If Q is the total volume of liquid flowing in t sec then $V = Q/t$.

$$\frac{Q}{t} = \frac{\pi h \rho g a^4}{8 l \eta}$$

$$\eta = \frac{\pi \rho g a^4 h t}{8 l Q}$$

If m is the mass of the liquid flowing in t sec & ρ its density then

$$Q = \frac{m}{\rho}$$

$$\eta = \frac{\pi \rho g a^4 h t}{8 l (m/\rho)}$$

$$= \frac{\pi \rho^2 g a^4}{8 l}\left(\frac{ht}{m}\right)$$

$$= \frac{\pi \rho^2 g a^4}{8 l}\left(\frac{ht}{m}\right) \quad \text{--- (8)}$$

Bernoulli's theorem:

Streamlined motion:

when the flow of a liquid through a tube is steady and smooth, every particle of the liquid passing through a given point follows the same path & has the same velocity as its predecessor.

In this motion, the volume of liquid flowing through every cross section of the tube per sec is the same.

vol. flowing / sec = Area of the section × velocity of flow

a_1 & a_2 → areas of cross section &

v_1 & v_2 → velocities,

$$a_1 v_1 = a_2 v_2$$

(ie) velocity of flow varies inversely as the area of cross section of the tube.

Bernoulli's theorem:

A liquid in streamline flow has the following three types of energy.

(i) K.E $= \frac{1}{2} mv^2$

K.E / mass $= \frac{1}{2} v^2$ [$\because m = 1$]

(ii) P.E $= mgh$

P.E / mass $= gh$

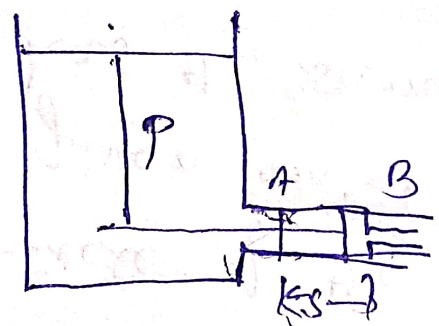
(iii) Pressure energy:

ρ - density of liquid

a - area of c.s.

Let a piston move in side tube.

P - pressure due to the liquid at level of the side tube.



Pa - force acts on the piston

s - distance

Pas - work done (or) energy of the mass $s a \rho$

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This energy possessed by the liquid at pressure P is called its pressure energy.

$\therefore$ Pressure energy / mass $= \frac{Pas}{sa\rho} = \frac{P}{\rho}$

This theorem states that in the streamlined flow of a liquid, the total energy of a certain mass of liquid of flowing from one point to another remain const. throughout.

$$hg + \frac{1}{2}v^2 + \frac{P}{\rho} = k \quad \text{—— (1)}$$

$$h + \frac{1}{2}\frac{v^2}{g} + \frac{P}{\rho g} = c \quad \text{—— (2)}$$

c - another const.

h - gravitational head

$\frac{1}{2}\frac{v^2}{g}$ - velocity head

$\frac{P}{\rho g}$ - Pressure head

This theorem may also ~~defined as~~ be stated as:

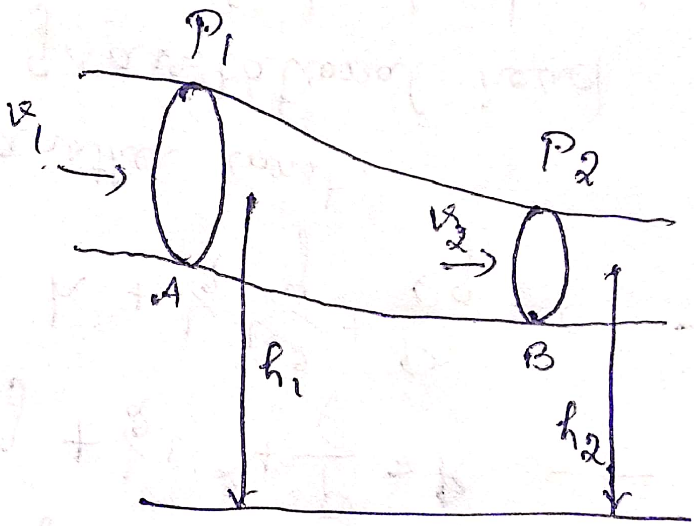
In the streamlined flow of a liquid from one point to another, the sum of the gravitational head, the velocity head and the pressure head remains const. throughout the displacement.

Proof:

consider a liquid flowing steadily through a non-uniform tube.

Let v_1 be velocity of flow at A of area of cross section a_1

Let P_1 be the pressure at A due to the driving pressure head.



∵ the same volume of liquid flows per sec through the sections A & B.

$$a_1 v_1 = a_2 v_2$$

∵ $a_1 > a_2$, $v_1 > v_2$

from A → B through distances $v_1 dt$ & $v_2 dt$.

∴ work done on the liquid at A

= Force at A × distance moved by liquid

$W_1 = P_1 a_1 \times v_1 dt$

$W_2 = P_2 a_1 v_1 dt$ $\quad [\because a_2 v_2 = a_1 v_1]$

The net workdone on the liquid in time dt by the pressure forces

$= W_1 - W_2$

$= (P_1 - P_2) a_1 v_1 dt$

ρ - density of liquid.

The same mass of liquid

$a_1 v_1 dt \rho = a_2 v_2 dt \rho$. moves through A & B. at dt

P.E of liquid at A $= (a_1 v_1 dt \rho) g h_1$

B $= (a_2 v_2 dt \rho) g h_2$

$= (a_1 v_1 dt \rho) g h_2$

$\therefore$ Increase in P.E $= (a_1 v_1 dt \rho) g (h_2 - h_1)$

K.E $= \frac{1}{2} (a_1 v_1 dt \rho) v_1^2$

" $= \frac{1}{2} (a_2 v_2 dt \rho) v_2^2$

$= \frac{1}{2} (a_1 v_1 dt \rho) v_2^2$

$\therefore$ Increase in K.E $= \frac{1}{2} (a_1 v_1 dt \rho) (v_2^2 - v_1^2)$

By the law of conservation of energy

$W_1 - W_2$ = Increase in P.E + Increase in K.E

$$(P_1 - P_2) a_1 v_1 dt = (a_1 v_1 dt \rho) g (h_2 - h_1) + \frac{1}{2}(a_1 v_1 dt \rho)(v_2^2 - v_1^2)$$

(ie) $$P_1 - P_2 = \rho g (h_2 - h_1) + \frac{1}{2}\rho (v_2^2 - v_1^2)$$

$$\rho g h_1 + \frac{1}{2}\rho v_1^2 + P_1 = \rho g h_2 + \frac{1}{2}\rho v_2^2 + P_2$$

$$\therefore \rho g h + \frac{1}{2}\rho v^2 + P = \text{a const}$$

[$\because a_1 v_1 = a_2 v_2$ – streamlined motion]

Div. by ρ, we get

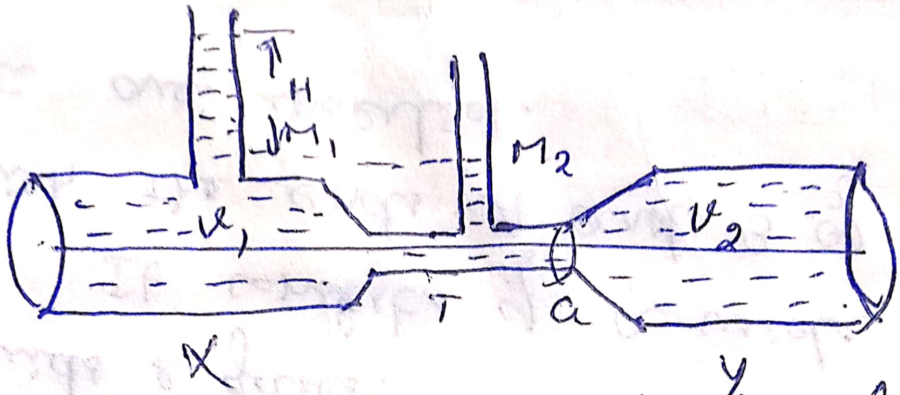
(or) $$gh + \frac{1}{2}v^2 + \frac{P}{\rho} = \text{const}.$$

[Bernouillie's theorem]

Venturimeter :-

Device – to measure the rate of flow of liquid in pipes

X & Y – tubes having same area A with constriction T. [throat]



when the flow is steady, let V be the flow is steady, let V be the volume of liquid flowing / sec through the venturi meter.

$$V = Av_1 = av_2$$

$$\therefore v_1 = \frac{V}{A} ; \quad v_2 = \frac{V}{a} \qquad \text{——(1)}$$

Applying Bernoulli's theorem to the flow in the wide portion X, Y & the throat T.

$$h_1 g + \frac{1}{2} v_1^2 + \frac{P_1}{\rho} = h_2 \rho + \frac{1}{2} v_2^2 + \frac{P_2}{\rho} \qquad \text{——(2)}$$

h_1 & h_2 - heights of liquids in X & T from the ground level,

ρ - density

$\because$ venturimeter is horizontal, $h_1 = h_2$

Hence eqn (2) becomes,

$$\frac{1}{2} v_1^2 + \frac{P_1}{\rho} = \frac{1}{2} v_2^2 + \frac{P_2}{\rho}$$

$$\therefore \frac{1}{2}(v_2^2 - v_1^2) = \frac{1}{\rho}(P_1 - P_2) \qquad \text{——(3)}$$

Subs. the values of v_1 & v_2 from eqn (1) ~~& (3)~~, in (3),

$$\frac{1}{2} v^2 \left(\frac{1}{a^2} - \frac{1}{A^2}\right] = \frac{1}{\rho}(P_1 - P_2)$$

$$\because P_1 - P_2 = H\rho g$$

where H - difference in levels in the venturimeter.

$$\frac{1}{2} V^2 \left[\frac{A^2 - a^2}{A^2 a^2}\right] = Hg$$

$$V^2 = A^2 a^2 \frac{2Hg}{[A^2 - a^2]}$$

$$V = Aa\sqrt{\frac{2Hg}{(A^2 - a^2)}}$$

(ie) $V \propto \sqrt{H}$ $\because$ A, a & g - consts.

Pitot tube:

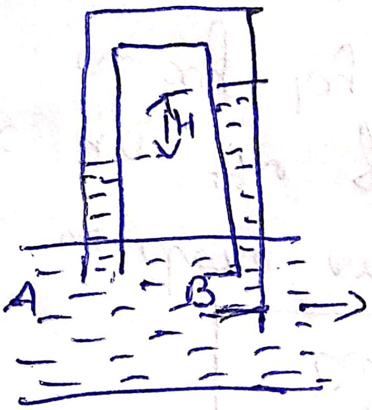
To measure the velocity of fluids in pipelines.

There are separate pitot's tubes for liquids & gases.

It consists of a wide pipe into which the ends A and B of manometer are inserted.

The plane of aperture of one of the ends A is $\parallel^l$ to the direction of flow of the liquid. The aperture at the end B faces the flow of fluid $\perp^r l_y$

P_1 & $P_2 \rightarrow$ pressure of fluid at A & B.



Applying Bernoulli's theorem to the ends A & B.

$$\frac{1}{2}v^2 + \frac{P_1}{\rho} = \frac{P_2}{\rho}$$

[$\because v = 0$ at B & gh is same at A & B]

$$v^2 = \frac{2}{\rho}[P_2 - P_1]$$

$$\therefore v = \sqrt{\frac{2}{\rho}(P_2 - P_1)}$$

For liquids, $P_2 - P_1 = H\rho g$.

$$\therefore v = \sqrt{2gH}$$

For gases, $P_2 - P_1 = Hdg \Rightarrow \therefore v = \sqrt{\left[\frac{2}{\rho}\right] Hdg}$

d - density of manometric liquid

$v \propto \sqrt{H}$

Surface tension: A liquid must experience some kind of force so as to occupy a min. surface area. This contracting tendency of a liquid surface is known as surface tension.

When a camel hair brush is dipped into water, the bristles spread out. When the brush is taken out, the bristles cling together on account of the films of water between them contracting.

(ie) The surface of a liquid behaves like an elastic membreane under tension (or) pull, with a tendency to contract.

Definition: It is the force / unit length of a line drawn in the liquid surface, acting $\perp^r$ to it at every point & tending to pull the surface apart along the line.

Unit: N/m

Dimension: $\frac{MLT^{-2}}{L} = MT^{-2}$

Molecular forces:

1. adhesive forces
2. cohesive forces

1. Forces of attraction between molecules of different substances.

Ex: force of attraction between the glass molecules of a beaker and molecules of water contained in it.

It is different for different pairs of substances.

2. Forces of attraction between molecules of same substances.

This force varies inversely probably as eight power of the distance between two molecules.

when the distance is small, it is very appreciable.

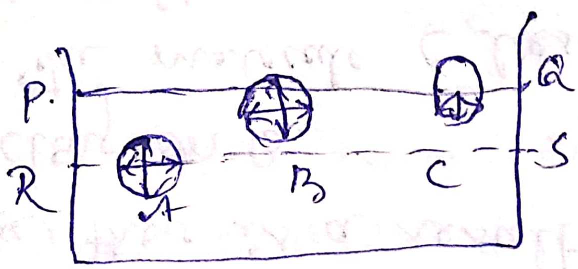
It is greatest in solids, less in liquids & least in gases.

∴ solid has a definite shape, a liquid has a definite free surface & gas neither.

The max. distance upto which a molecule exerts a ~~free~~ force of attraction on another is called the range of molecular attraction ($10^{-9}m$)

A sphere with the molecule as centre and the range of ~~a~~ molecular attraction as radius → sphere of influence.

Explanation of Surface tension on Kinetic Theory



A, B, C - molecules

(i) A is attracted equally in all directions by the other molecules lying within its sphere of influence (ie) it does not experience any resultant force in any direction.

(ii) ~~At~~ B lies partly out-side the liquid. The upper half of the sphere contains fewer molecules attracting the molecule B upwards, than the lower half attracting it downwards.

ie there is a resultant downward force acting on B.

(iii) The molecule C lies on the surface of the liquid. Half of its sphere of influence lies

upper half - few vapour molecules

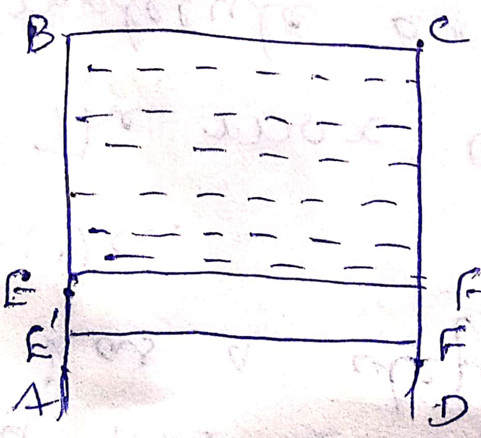
Lower half - many no. of liquid molecule

∴ the resultant downward force in this case is the maximum.

~~Pa~~ The planes PQ & RS - Surface film.

All the molecules in the surface film are pulled downward due to the cohesive force between molecules.

<u>work done in increasing the area of a surface:</u>



ABCD - rectangular framework of wire

(S.T) σ - force / unit length of the film (S.T

l - length of the wire EF

upward force due to S.T

$= 2l\sigma$

∵ The film has two surfaces & each has a S.T σ]

∴ $F = 2l.\sigma$

If the wire is pulled downwards through a small distance x to the position E'F'

∴ work done $= Fx$

$= 2l\sigma.x$

$= \sigma.2l.x$

$=$ S.T × Increase in surface area

∴ work done in increasing the surface area of the liquid film by unity $= \frac{\sigma 2lx}{2l.x} = \sigma$

∴ S.T may be defined as the amount of workdone in increasing the surface area of the liquid film by unity.

Work done in blowing a bubble:

Let radius of the bubble blown be r.
A bubble has two surfaces, an inner & an outer one, each of surface area $4\pi r^2$

∴ ~~S.T~~ Surface area of the film forming the bubble $= 2\times 4\pi r^2 = 8\pi r^2$

∴ workdone in blowing the bubble

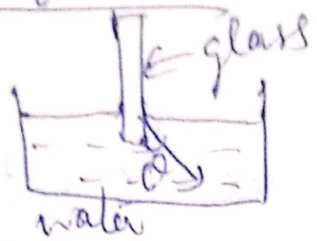
= S.T × surface area of the film formed.

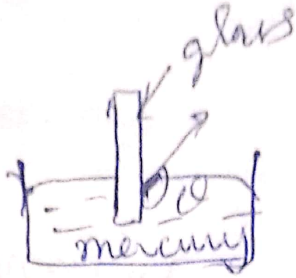
$= \sigma \times 8\pi r^2 = 8\pi r^2\sigma$

Forms of liquid drops:

when liquid rests upon a horizonal solid plate, which it does not wet, the shape of the drop is determined by surface tension and gravity. For extremely small drops, the S.T effects are great & the gravitational effects small. So S.T determines the shape of the drop. (ie) it is spherical

Angle of contact:





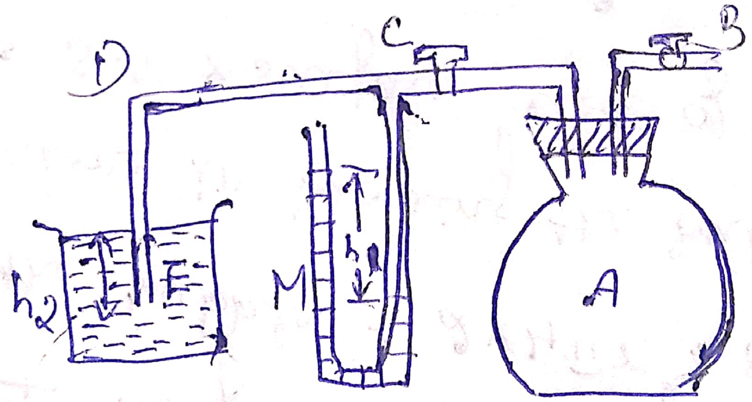
— x —

Jaegar's method:

It is based on the principle that the pressure inside an air bubble in a liquid is greater than the pressure outside it by $2\sigma/r$.

σ - S.T

r - radius of the air bubble



The pressure inside the bubble $= P_1$

$= H + h_1 \rho_1 g$

H - atmospheric pressure,

h_1 - diff. in manometer levels

ρ_1 - density of the manometric liquid

The pressure outside the bubble at the same time $\} \therefore P_2 = H + h_2 \rho_2 g$

h_2 - Length of the tube dipping in the experimental liquid

ρ_2 - Density of the exp. liquid

Excess pressure inside the bubble $\} = p = (H + h_1 \rho g) - (H + h_2 \rho_2 g)$

$= (h_1 \rho_1 - h_2 \rho_2) g$

But the excess pressure inside the bubble $= 2\sigma / r$

$2\sigma / r = (h_1 \rho_1 - h_2 \rho_2) g$

$\therefore \sigma = \frac{1}{2} rg (h_1 \rho_1 - h_2 \rho_2)$

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